A MODELING STUDY OF THE PIN-ASSISTED RESIN INFILTRATION OF POROUS SUBSTRATES

Nickolas D. Polychronopoulos, T.D. Papathanasiou
Department of Mechanical Engineering, University of Thessaly, 38334 Volos

ABSTRACT
We report on the results of a two dimensional numerical study using OpenFOAM, of the process of infiltration of a homogeneous porous substrate, moving over a stationary solid cylinder, by an incompressible, isothermal and Newtonian fluid. The motion of the substrate relative to the cylinder causes a pressure build-up in the wedge-shaped region separating them, which forces the fluid to penetrate into the porous material. The analysis assumes Stokes flow outside the substrate and uses Brinkman's equation to describe the flow inside the substrate. The structure of the flow field is characterized by the combination of a drag and an opposing pressure-driven flow, the latter caused by the above mentioned pressure build-up in the wedge. The amount of fluid penetrating into the substrate is found to be affected by various parameters such as the pulling speed, substrate permeability, cylinder diameter and substrate thickness. Based on large numbers of simulations, we correlate the achieved impregnation depth to the above parameters and propose a previously unavailable quantitative relationships expressing the above dependencies. Our results find direct applicability in the modeling of the pin-assisted pultrusion process and are in good agreement with the limited experimental data available in the technical literature.

INTRODUCTION
The pultrusion process is a manufacturing technology in which reinforcing fibers impregnated with polymer are pulled through a die to consolidate and shape the final composite product. The infiltration of the polymer into the fibrous porous network is a key step in these technologies and quite often it is linked to the quality and final performance of the product. Poor or incomplete impregnation of the fibrous substrate often leads to failure of the composite components. To facilitate this, besides control of the architecture of the fibrous preform/substrate, pressure is applied. This can take several forms [1-3]. In the pultrusion process of interest to this study, namely the pin-assisted pultrusion, bundles of fibers (which usually come in the form of a flexible roving) are pulled through a die and over an array of cylindrical solid pins by a wrap angle located inside a non-pressurized melt pool [4,5]. During the pulling of the porous roving around each pin a small wedge-shaped region of polymeric resin is formed between the pin's surface and the roving. At this region the generated pressure caused by the dragging forces the resin to penetrate the porous bundle while the fibers consisting it laterally spread. Due to this fiber rearrangement the flow in the porous substrate is redistributed and more impregnation can take place when systems of multiple pin arrangements are utilized.

The studies regarding the modelling of pin-assisted pultrusion are very limited in number [6-11]. Their main finding is underlined by the presence of a large number of parameters (such the pulling speed, permeability of the porous substrate, pin diameter and substrate thickness) the range of which seems to affect the fluid infiltration to a large extend. In contrast to [6-11] where 1D formulations were carried out, the present authors in [12] utilized a 2D framework by Stokes equation to describe the free fluid flow and Brinkman's equation for the porous flow of a single pin process. This allowed to express the amount of infiltration as a function of dimensionless groups the presence of which aids in giving a semi-generic view of the process. Motivated by this, in the present study, we seek for a universal generic correlation of the fluid infiltration with the process parameters, which yet has not been reported in the literature for the specific process. This may be performed if all the process parameters are grouped together into a single universal dimensionless group of substantial physical meaning which, as it will be explained, produces data that fall into the same curve with a relatively small scatter. Utilizing such a correlation allowed us to make predictions for a sequential arrangement of pins with good overall agreement with some experimental results available from the literature.

TWO DIMENSIONAL MODEL AND BOUNDARY CONDITIONS
The wrapping of the fiber bundle over the pin leads to the formation of three distinct regions between the each latter and the former: (a) a converging wedge-shaped region, the length of which extends from the maximum distance between the pin and the roving to the zero tangency point (i.e. the minimum distance of pin-roving), (b) a consecutive region of non uniform thickness (the effective wrap angle region) at which the roving is partially in contact with the pin and (c) a final diverging wedge-shaped region in which the roving separates from the pin.
The present analysis considers a single pin process in which the resin infiltration is assumed to occur only in region (a). This ignores any additional infiltration that may occur in zone (b). This is a reasonable assumption, since the extend of this zone is unknown, avoiding the need to insert arbitrary variables in the analysis. This approach is in line with earlier work on the topic [11, 12]. As assumed in [12] we also here take full account the presence of a large resin pool compared to the dimensions of the pin. In [10] it is reported that the fibrous roving may partially be in contact with the pin which results in possible build up of frictional forces. The present analysis ignores any such phenomena, mainly because of the inability to measure the relevant geometrical parameters. It is also known that as the fibrous roving is dragged over the pin there is a possible lateral spread over the pin’s entire length accompanied with lateral leakage flow. As reported in [6] this is a result by utilizing non cylindrical pins (of convex or concave shape with respect to their symmetry axis). This spreading is beyond the scope of the present study.

The clear fluid located in the pool is assumed to be incompressible, isothermal, Newtonian and the flow is modeled by the Stokes equation

\[ 0 = \nabla p + \mu \nabla^2 \mathbf{U} \quad (1) \]

where \( p \) the pressure, \( \mathbf{U} = (u_x, u_y) \) the velocity vector and \( \mu \) the viscosity of the fluid, combined with the incompressibility condition \( \nabla \cdot \mathbf{U} = 0 \). In the area of composites this is justifiable in light of the low elasticity/low processing speed and high viscosity of the resins typically used. The fibrous roving is assumed to be an isotropic homogeneous porous medium the flow in which is described by Brinkman’s equation

\[ \nabla p = -\mu K^{-1} \mathbf{U} + \mu_e \nabla^2 \mathbf{U} \quad (2) \]

where \( K \) the permeability tensor (which is scalar for isotropic porous media) of the medium, \( \mu \) the clear fluid viscosity and \( \mu_e \) the effective viscosity.

The geometry utilized is shown in Figure 1a. A pre-impregnated zone of \( L_o \) thickness is assumed to exist the value of which may vary. This zone moves with speed \( V \) (pulling speed) in the x-direction. The pool where the clear fluid is situated is confined by the lengths \( l_x \) and \( l_y \) which are defined as \( l_x = l_y > R \). In this way, the boundary conditions imposed on the corresponding boundaries (boundaries FG and GH) will have little, if any, effect to the behaviour of the flow in the formed wedge-shaped region. The depth of the fluid’s penetration into the medium (which is a parameter determined at the end of the computation) is denoted as \( h_f \) and \( \delta \) the separation height between the pin’s surface and the medium. The constructed two-dimensional computational domain (before the computation of \( h_f \)) for \( L_o = 4 \) mm, \( \delta = 0.5 \) mm and \( R = 5 \) mm is shown in Figure 1b.

It should be pointed out that consideration of a pre-impregnated zone is vital so that a continuum model for the process can be established. Neglecting the inclusion of such a zone in the current formulation would necessitate to regard the dry substrate as a two phase region in which air entrapments would alternate with the

**Figure 1.** (a) Schematic representation of the model geometry for impregnated fibrous substrate. \( R \) is the pin radius, \( L_o \) the thickness of the pre-impregnated porous zone, \( \delta \) the separation height between the porous and the pin, \( h_f \) the infiltration depth, \( L_o \) corresponds to the length of the formed wedge-shaped region between the substrate and the pin and \( l_x, l_y \) the distances extending the pool of the clear fluid. The porous substrate moves with speed \( V \). (b) Sample computational domain for \( R = 5 \) mm, \( L_o = 4 \) mm, \( \delta = 5 \) mm.
solid fibrous network, and which on one side will be in contact with the clear fluid. In earlier works, such as in [11] this physical picture of the process is implicit, though not utilized. It has been used with success by Chen and Papathanasiou [13] for modeling the flow through fully saturated fibrous media and by Tan et al [14] to model the flow at the interface between a saturated fiber array and a clear fluid. However, treating the problem at hand as one of unsaturated multiphase flow, would necessitate the use of a free surface tracking scheme in a geometrically complex, unknown a priori, multiply-connected domain. While this is in principle feasible, the complexities involved in free surface tracking as well as in describing realistically the internal geometry of the substrate would divert from the present task.

With reference to Figure 1a, and for the pool where the clear fluid is located the following boundary conditions apply

- Boundaries EF, FG and GH: we set $\partial U/\partial n$, for the velocity and $p=0$ (ambient) for the pressure, reflecting the fact that these boundaries are located far away from the wedge-shaped region. $n$ is the unit normal vector of each boundary.
- Boundary DE (pin’s surface): no-slip condition for velocity and $\partial p/\partial n = 0$ for the pressure.
- Boundary CD: we set $\partial p/\partial n = 0$ and $\partial U/\partial n = 0$. These conditions imply the presence of a constant separation zone past the zero tangency point, of uniform thickness, as argued above.

For the homogeneous porous substrate we set the following boundary conditions

- Pre-impregnation zone (Figure 1) is assumed to move with speed $V$.
- Boundary BC: similar to the boundary CD (see above).
- Boundary AH: moves with speed $V$ ($U=V$).
- Boundary AB: This boundary represents the resin profile within the porous medium (see Figure 1a) whose shape is a priori unknown and is determined in the course of the computation. By applying $\partial U/\partial n = 0$ and $p=0$ we assume that the substrate is sufficiently thick. In other words, we do not consider the case of a thin substrate that can be fully impregnated before a pass around the pin is completed. Following the iterative procedure described in the next section for the determination of line AB shape, the velocities on that line, after convergence will satisfy $U \cdot n = 0$.

It should be noted that when the Brinkman equation (Eq. 2) is used to model the porous flow, together with Stokes equation (Eq. 1) for the clear fluid, one of the most common ways of dealing with the boundary conditions at the interface is that both the velocity and stress are assumed to be continuous [15]. For the solution of the governing equations (continuity equation along with the Stokes and Brinkman equations) we use the OpenFOAM (Open Source Field Orientation and Manipulation) package utilizing the Finite Volume Method (FVM). In all the simulations performed, the domain was subdivided using an approximate total of $3\times10^4$ cells. Grading of the mesh density close to the formed wedge-shaped region was also employed.

**PREDICTION OF FLUID PENETRATION PROFILE**

In the present section we describe the predictor-corrector method used to predict the shape of the boundary AB in Figure 1 and thus the fluid’s infiltration depth $h$. A similar form of this method was used in [16] to determine the spreading of a sheet passing between two rotating cylinders. The final shape of AB will be determined by requiring that at steady state the fluid velocity $u_y = u_x \cos \theta - u_x \sin \theta$ across that boundary is zero. This results

$$\tan \theta \equiv \frac{dy}{dx} = \frac{u_x}{u_y}$$

(3)

where $dy/dx$ is the local slope of line AB, $u_x$ is the velocity in the transverse direction ($y$-direction) and $u_y$ the velocity in the $x$-direction. To determine the shape of the infiltration profile, the continuity, Stokes and Brinkman equations are solved for an initial configuration, for example with AB as a straight line. At the end of this step (predictor), the velocity and pressure fields are known everywhere, including on line AB. The corrector step involves the correction of the shape of boundary AB, based on integration of Eq. 4, namely

$$y_j^{n+1} = y_j^n + \sum_{j=1}^{N_i} \left( \frac{u_y}{u_x} \right)_j \left( x_j^n - x_{j-1}^n \right)$$

(4)

In Eq. 5 $y_j$ corresponds to the updated $y$-coordinate at the corresponding nodal point at the $n^{th}$ iteration, $y_{o}$ is the $y$-coordinate of point A (at $x=0$) in Figure 1, $x_{j-1}$, the distance between two successive nodal points of boundary AB in the $x$-direction and $N_i$ the number of nodes in the $x$-direction. Having determined a new shape of
Figure 2. Infiltration profiles for $L_o=2$ mm, $R=5$ mm, $V=0.15$ m/s and $\mu=1000$ Pa·s for a distance $x=2R$. To the right of the vertical dashed line corresponds the region for exactly one pin radius.

![Infiltration profiles](image)

boundary AB and thus a new computational domain, a new mesh is constructed and the Stokes, Brinkman and continuity equations are solved again. The new velocity field is used to correct the shape of the boundary AB through Eq. 5. The procedure continues until the normal velocity across boundary AB, as determined by the predictor step, is less than a prescribed tolerance ($\varepsilon=10^{-6}$), namely

$$\frac{1}{N_x} \sum_{j=1}^{N_x} u_{nj}^2 \leq \varepsilon$$

RESULTS AND DISCUSSION

In Figure 2 the infiltration profiles are plotted based on the algorithmic procedure described on an earlier section for a permeability range of $K=10^{-7} \sim 10^{-9}$ m$^2$. Both axes are non-dimensionalized with the length $l_x$ of the computational domain (see Figure 1). The vertical distance of all the computed curves from the non-impregnated

Figure 3. Different directions on which the $(u_x, u_y)$ velocity components are plotted. (a) corresponds for $u_x$ and (b) for $u_y$ (note: $y_j=\delta/2$).
straight line, corresponds to the penetration depth $h_f$. Increasing the permeability of the porous substrate facilitates the infiltration process of the fluid. It is interesting to observe that for the case of a relatively permeable substrate the presence (i.e. $K=10^{-7}$ m$^2$) of the pin causes a large disturbance in the fluid's flow field which results in an early impregnation of the substrate well ahead of the pin.

Some of the most fundamental fluid mechanics of the process are elucidated. The structure of the flow is described by calculation of velocity profiles at various locations in the $y$- and $x$-directions shown in Figure 3a and Figure 3b respectively. For the locations of Figure 3a the $u_x$ velocity profiles are shown in Figure 3c. For the location $x=x_4$ the shape of the velocity profile indicates a back flow. This effect is also present for regions located far behind the wedge-shaped region. At the intermediate locations in this wedge (i.e. $x=x_2$ and $x=x_3$) the back flow is less intense and the fluid begins to locally move via simple shear which is, more pronounced close to the zero tangency point at $x=x_1$. These observations imply that the local flow field in the wedge is the result of the combined action of a pressure-driven back-flow caused by the pressure build up within the wedge and the drag flow caused by the pulling of the porous substrate. The shape of the above mentioned profiles, and for regions inside the porous In Figure 3d the $u_y$ velocity profiles are shown for the locations of Figure 3b for a distance of $x=2R$. For $y=y_1$ there are negative velocity values which reflect the two-dimensional nature of the flow while positive values predominate near the zero tangency-point which explain the fluid's infiltration into the substrate. Within the substrate only positive $u_y$ velocity values are observed. These results suggest that there is a significant amount of fluid that moves away from the porous substrate and the pin as a result of the pressure-driven back flow. For this reason, not all fluid dragged by the substrate will approach the pin and even less will penetrate into the substrate. Of course, the relative amounts depend primarily on the permeability of the substrate as more permeable media imply ease of fluid penetration and therefore low tendency for back flow.

In a previous study [12] it was rigorously elucidated that the penetration depth may be affected by different operating and geometric parameters expressed via dimensionless quantities such as, $\delta_oL_o\mu$, $L_o/\sqrt{K}$ and $L_o/R$ for a single pin process. Such a scaling procedure was developed by examining separately each time a specific parameter and the suggested scaling was a direct and natural consequence of raw computational data obtained. A very significant parameter which was neglected in the correlation of the obtained numerical data is the generated pressure $P$ in the wedge shaped region formed between the pin and the substrate. It is convenient to express this pressure in terms of a average generated pressure $P_{av}$ as

$$P_{av} = \frac{1}{L_o} \int P(x)dx \quad (6)$$

assuming that this average pressure is applied over a distance $L_o=R$. We perform simulations for a wide range of process parameters ($\mu, V, L_o, \delta_o, K, R$), and for each produced penetration depth $h_f$ we calculate the average pressure $P_{av}$ based on Equation 6. Based on the dimensionless scaling arguments discussed previously, the above-mentioned process parameters may be grouped together into a single universal dimensionless parameter as

Figure 4. Effect of the global dimensionless parameter on the penetration depth $h_f$. 

![Figure 4](image-url)
(\mu L_o V/KP_{\infty})(\delta_j/R). By doing so and as shown in Figure 4 the obtained data appear to produce a generic trend as they into the same region which a relatively small scatter. This trend may be sufficiently be expressed by fitting the produced data as

\[ h_f = \frac{A}{1 + B \left( \frac{\mu L_o \delta \omega}{P_{\infty} K R} \right)^C} \]

where \(A = 8.21 \times 10^{-4}\), \(B = 0.879\) and \(C = 1.2\) are fitted parameters. The global curve expressed by Equation (7) may give an estimate of the penetration depth at distinguishably different types of conditions by which impregnation may take place. Essentially, the global dimensionless group could be regarded as the ratio of viscous resistance of fluid flow to the fluid penetration by permeation multiplied by a scaled form the pin radius which expresses the geometric parameters of the process. When the viscous forces dominate the flow pertrusion by permeation is small while the opposite takes place when the viscous forces are low. For cases where the viscous force is balanced by the fluid penetration due to permeation increasing the pin radius will increase the fluid infiltration into the porous. Moreover, it can be concluded that the fluid viscosity, production rate (i.e. pulling speed \(V\)), permeability of the substrate and pin radius can be adjusted accordingly for the production of similar quality impregnated fibrous products. It should also be pointed out that the finite value \(A = 8.21 \times 10^{-4}\) of Equation 8 corresponds to the limit of a dry substrate. Our results indicate that at this limiting case the infiltration depth is essentially not affected by the process conditions. While this is not the case for the real pin-assisted process we assume that at this limiting condition the infiltration depth remains constant as an initial approximation for the subsequent study of multi-pin arrangements at which the substrate passing over the next pin is not dry.

It would be tempting to use Equation 7 above to evaluate the fluid penetration for sequential arrangement of pins and under certain operating conditions. The presence of the \(L_o\) variable may be regarded as a multi-pin parameter which aids in expressing the penetration depth for each pin number if used in an iterative manner. By this formulation, for the next pin the thickness of the pre-impregnated zone will be equal to the infiltrated depth \(h_f + L_o\) of the previous pin. Then the final infiltration depth \(L_f\) on each pin will be simply

\[ L_f^N = L_o^{N-1} + h_f^{N-1} \]

where \(N\) corresponds to the pin number. For each consecutive pin the infiltration depth \(h_f\) may then be estimated by introducing Equation (7) to Equation (8). However, in Equation (8) the average generated pressure \(P_{\infty}\) may frequently be unknown as it will vary from the first pin to the consecutive one for a specified number of pins. This can be avoided if the average pressure is expressed in terms of another related-to-pressure parameter which for a specific pin arrangement and under specified operating conditions \(V, R\) as well as properties of the fluid \(\mu\) and the substrate \(K\), can be approximated to be constant. By such, a simple force balance implies that pulling tension \(T\) may qualify as a constant parameter under specific conditions. If we assume that the applied tension \(T\) is applied normal to the substrate cross-sectional area defined as \(WL_o\), where \(W\) is the substrate width over the pin, then the average pressure may be expressed as \(P_{\infty} = T/WL_o\). Then by substituting in Equation 7 and the obtained equation to Equation 8 gives us an estimate of the penetration depth for a multi-pin arrangement as

\[ L_f^N = L_o^{N-1} + \left( \frac{8.21 \times 10^{-4}}{1 + 0.879 \left( \frac{\mu L_o^2 W \delta \omega}{TK R} \right)^{1.19}} \right)^{N-1} \]

where \(N\) is the number of pins utilized. The behaviour of Equation 9 was also compared with available experimental results from the literature as shown in Figure 5. In Figure 5a the effect of the pin number \(N\) on the infiltration degree \(D_t\) under different pin radius \(R\) is shown. The symbols correspond to the experimental results as reproduced from Bates and Zou [11] for \(V = 0.1833\) m/s and the dashed lines express the present predictions of Equation (11) for the same pulling speed. We have also assumed in our calculations \(T = 20\) N, \(\mu = 100\) Pa·s, \(W = 10\) mm and \(\delta = 0.5\) mm. These assumed values are relative to the pin-assisted impregnation process [8-10]. Another assumption made for the comparison was that for \(R = 5\) and \(7\) mm the substrate was fully impregnated for \(N = 5\)
**REFERENCES**